which equals the square of the *difference* of the two roots, or

$$\left(2n+1+2\sqrt{\frac{n(n+1)}{2}}\right)^2$$

Illustration.—From the series of triangular square numbers, 1^2, 6^2, 35^2, 204^2, 1189^2, etc., take 6 and 35. $35-6=29$; $35+6=41=20+21$; $20^2+21^2=29^2$.

This problem and problems No. 45, (Vol. III., No. 5, page 153), and No. 36, of Diophantine Analysis, are very closely related.

Also solved by the *PROPOSER*.

52. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics in Columbiam University, Washington, D. C.

Prove that a "magic square" of nine integral elements, whose rows, columns, and diagonals have a constant sum, is only possible when this sum is a multiple of three.

I. Solution by M. W. HASKELL, M. A., Ph. D., Associate Professor of Mathematics, University of California, Berkeley, California.

Let the magic square be

a	b	c
d	e	f
g	h	k

and let S be the constant sum.

Then $S=a+b+c=d+e+f=g+h+k=a+d+g=b+e+h=c+f+k=a+e+k=c+e+g$.

Adding these all together, we have $8S=3a+2b+3c+2d+4e+2f+3g+2h+3k=3(a+c+g+k)+2(b+e+h)+2(d+e+f)$. But the last two quantities in parenthesis are each$=S$. Hence $4S=3(a+c+g+k)$, and S is a multiple of 3.

II. Solution by —— (Paper Unsigned.)

Suppose the numbers occupying the magic square to be a, b, c, d, e, f, g, h, k. Now $a+e+k=b+e+h=c+e+g=S$.

$\therefore$ $a+k\equiv k(\text{mod } 3)$, $b+h\equiv k(\text{mod } 3)$, $c+g\equiv k(\text{mod } 3)$, where $S-e\equiv k(\text{mod } 3)$.

Adding the congruences, $(a+b+c)+(g+h+k)\equiv 0(\text{mod } 3)$. Or, since $(a+b+c)+(g+h+k)\equiv 0(\text{mod } 3)$, $2S\equiv 0(\text{mod } 3)$.

Multiply by 2, and divide by 3, and the result is $S\equiv 0$. Q. E. D.

III. Solution by W. H. CARTER, Professor of Mathematics, Centenary College of Louisiana, Jackson, Louisiana.

Let the rows of the "square" be a, b, c ; x, y, z ; and l, m, n, and let the constant sum be k. We have to show that $k/3$ is integral. We have $a+y+n=k$; $b+y+m=k$; $l+y+c=k$. Add, and we have $(a+b+c)+(l+m+n)+3y=3k$, that is, $2k+3y=3k$.

$\therefore$ $3y=k$. $\therefore$ $y=k/3$. But y is integral. $\therefore$ $k/3$ is integral.

Also solved by *M. A. GRUBER* and *G. B. M. ZERR*.